Reg. No. :

Question Paper Code : 91778

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/ DECEMBER 2019

First Semester

Mechanical Engineering

MA 6151 – MATHEMATICS – I

(Common to all Branches Except Marine Engineering)

(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Find the sum and product of all the eigen values of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

2. If $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ is the matrix of a quadratic form, find its nature.

3. Give an example for conditionally convergent series.

4. Test the convergence of the series $\sum \frac{1}{n^2+1}$.

5. Define evolute of a curve.

6. Find the envelope of the family of curves $y = mx + \frac{1}{m}$, where m is the parameter.

7. If $u = \sin^{-1}\left[\frac{x^3 - y^2}{x + y}\right]$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$.

8. Find $\frac{du}{dt}$, if $u = \frac{x}{y}$, where $x = e^t$, $y = \log t$.

9. Evaluate $\int_0^1\int_0^2\int_0^3 xyz\,dx\,dy\,dz$.

10. Change the order of integration in $\int_0^1\int_0^y f(x, y)\,dx\,dy$.

PART – B (5×16=80 Marks)

11. a) i) Find the eigenvalues and the eigenvectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. (8)

ii) Using Cayley-Hamilton theorem, find A^{-1} and A^4, if

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$ (8)

(OR)

b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. Hence, find its rank and nature. (16)

12. a) i) Discuss the convergence and the divergence of the following series.

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots \text{to } \infty.$$ (8)

ii) Find the interval of the convergence of the series.

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \ldots$$ (8)

(OR)

b) i) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{1}{n}\right)$. (8)

ii) Test the convergence of the series $\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \ldots \text{to } \infty$. (8)

13. a) i) Find the radius of curvature of $x^{2/3} + y^{2/3} = a^{2/3}$. (8)

ii) Obtain the evolute of $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. (8)

(OR)

b) i) Find the centre of curvature of $x^3 + y^3 = 6xy$ at (3, 3). (8)

ii) Obtain the envelope of $\frac{x}{a} + \frac{y}{b} = 1$, if $a^2 + b^2 = c^2$. (8)

14. a) i) A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box that requires the least material for its construction. (8)

ii) Find the minimum values of x^2yz^3 subject to the condition $2x + y + 3z = a$. (8)

(OR)

b) i) Obtain the Taylor series of $x^3 + y^3 + xy^2$ at (1, 2). (8)

ii) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-1}{2}\cot u$. (8)

15. a) i) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\, dx\, dy$. (8)

ii) Using double integral, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8)

(OR)

b) i) Change the order of integration in $\int_0^2 \int_0^{\sqrt{4-y^2}} xy\, dx\, dy$ and evaluate it. (8)

ii) By transforming into polar co-ordinates evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$. Hence find the value of $\int_0^\infty e^{-x^2} dx$. (8)
