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## Question Paper Code : 91778

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/ DECEMBER 2019  
First Semester

Mechanical Engineering  
MA 6151 – MATHEMATICS – I  
(Common to all Branches Except Marine Engineering)  
(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

### PART – A

(10×2=20 Marks)

1. Find the sum and product of all the eigen values of  $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ .
2. If  $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$  is the matrix of a quadratic form, find its nature.
3. Give an example for conditionally convergent series.
4. Test the convergence of the series  $\sum \frac{1}{n^2+1}$ .
5. Define evolute of a curve.
6. Find the envelope of the family of curves  $y = mx + \frac{1}{m}$ , where  $m$  is the parameter.
7. If  $u = \sin^{-1} \left[ \frac{x^3 - y^2}{x + y} \right]$ , then prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \tan u$ .
8. Find  $\frac{du}{dt}$ , if  $u = \frac{x}{y}$ , where  $x = e^t$ ,  $y = \log t$ .



9. Evaluate  $\iiint_{0 \ 0 \ 0}^{1 \ 2 \ 3} xyz \, dx \, dy \, dz$ .

10. Change the order of integration in  $\iint_{0 \ 0}^{1 \ y} f(x, y) \, dx \, dy$ .

## PART - B

(5×16=80 Marks)

11. a) i) Find the eigenvalues and the eigenvectors of the matrix  $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ . (8)

ii) Using Cayley-Hamilton theorem, find  $A^{-1}$  and  $A^4$ , if

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}. \quad (8)$$

(OR)

b) Reduce the quadratic form  $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$  into a canonical form by an orthogonal reduction. Hence, find its rank and nature. (16)

12. a) i) Discuss the convergence and the divergence of the following series.

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots \text{to } \infty. \quad (8)$$

ii) Find the interval of the convergence of the series.

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \dots \quad (8)$$

(OR)

b) i) Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{1}{n}\right)$ . (8)

ii) Test the convergence of the series  $\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \dots \text{to } \infty$ . (8)

13. a) i) Find the radius of curvature of  $x^{2/3} + y^{2/3} = a^{2/3}$ . (8)

ii) Obtain the evolute of  $x = a(\theta + \sin \theta)$ ,  $y = a(1 - \cos \theta)$ . (8)

(OR)

b) i) Find the centre of curvature of  $x^3 + y^3 = 6xy$  at (3, 3). (8)

ii) Obtain the envelope of  $\frac{x}{a} + \frac{y}{b} = 1$ , if  $a^2 + b^2 = c^2$ . (8)

14. a) i) A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box that requires the least material for its construction. (8)

ii) Find the minimum values of  $x^2yz^3$  subject to the condition  
 $2x + y + 3z = a$ . (8)

(OR)

b) i) Obtain the Taylor series of  $x^3 + y^3 + xy^2$  at (1, 2). (8)

ii) If  $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x+y}}\right)$ , then prove that  $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-1}{2}\cot u$ . (8)

15. a) i) Evaluate  $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2} dx dy$ . (8)

ii) Using double integral, find the area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . (8)

(OR)

b) i) Change the order of integration in  $\int_0^{2\sqrt{4-y^2}} \int_0^{xy} dx dy$  and evaluate it. (8)

ii) By transforming into polar co-ordinates evaluate  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ . Hence find the value of  $\int_0^\infty e^{-x^2} dx$ . (8)

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177 10.  $\partial_x^2 \partial_t u + \partial_x u - \frac{1}{2} u^2 + \frac{1}{2} u_0$  satisfies the initial condition (0)

$$178 \quad \partial_x u = \frac{\partial}{\partial x} u = \frac{\partial}{\partial x} u_0 + \frac{1}{2} u^2 + \frac{1}{2} u_0^2 \text{ to a positive admissible } u$$

179 will have to take into account a small initial speed only to prove real existence for  $u$ . If the initial function  $u_0$  has a jump discontinuity it is not enough to assume that the discontinuity is small.

180 Therefore we can choose  $u_0$  to satisfy conditions and hope to

$$181 \quad u = g(t) + V + \frac{1}{2} u^2$$

(10)

182  $\partial_x^2 \partial_t u + \partial_x u + \frac{1}{2} u^2$  is strong subadditive (0)

$$183 \quad \text{Let } \partial_x \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial x} \text{ satisfies the PDE } \left( \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{1}{2} u^2 \right) + \text{smooth } u_0 \text{ (0)}$$

184  $\partial_x \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial x}$  satisfies (0) in (0)

185  $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{1}{2} u^2$  satisfies subadditive with limit comparison global smooth (0)

(10)

186  $\partial_x^2 \partial_t u + \partial_x u + \frac{1}{2} u^2$  is strong subadditive (0)

187  $\partial_x^2 \partial_t u + \partial_x u + \frac{1}{2} u^2$  satisfies subadditive with limit comparison global smooth (0)

188  $\partial_x^2 \partial_t u + \partial_x u + \frac{1}{2} u^2$  satisfies (0) in (0)